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Generic implementation of semi-analytical CI gradients for NDDO-type methods

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Abstract Generic semi-analytical energy gradients are derived and implemented for NDDO-type methods, by using numerical integral and Fock matrix derivatives in the context of an otherwise analytical approach for configuration interaction (CI) and other non-variational treatments. The correctness, numerical precision, and performance of this hybrid approach are established through comparisons with fully numerical and fully analytical calculations. The semi-analytical evaluation of the CI gradient is generally much faster than the fully numerical computation, but somewhat slower than a fully analytical calculation, which however shows the same scaling behavior. It is the method of choice whenever a fully analytical CI gradient is not available due to the lack of analytical integral derivatives. The implementation is generic in the sense that it can easily be extended to any new NDDO-type Hamiltonian. The present development of a semi-analytical CI gradient will facilitate studies of electronically excited states with recently proposed NDDO methods that include orthogonalization corrections.

1 Introduction

The established NDDO-type semiempirical methods MNDO [1], AM1 [2], and PM3 [3] have been applied extensively to study the ground-state properties and reactions of large molecules [4–6]. These methods possess a very simple and regular structure of the one-electron Hamiltonian and the two-electron integrals, which facilitates analytical evaluation of energy derivatives. As a result, energy gradients can be

computed in a computationally very efficient manner for both variational [7, 8] and non-variational [8–10] wavefunctions, based on these Hamiltonians.

The more recent NDDO-type approaches with orthogonalization corrections (OM1 [11, 12], OM2 [13, 14], and OM3 [15]) have been designed to overcome some of the well-documented deficiencies [4–6] of the older methods. These approaches have significantly improved the description of conformational barriers and electronically excited states. However, these advances come at the cost of more intricate expressions for both the one-electron Hamiltonian and the two-electron integrals. This complexity makes a fully analytical gradient implementation in OMx methods a daunting and error-prone task.

Computationally efficient, semi-analytical energy gradients of variational OMx self-consistent field (SCF) energies are available [13]: they are computed from the given constant density matrix and numerical integral derivatives. However, in all non-variational treatments, including half-electron open-shell Hartree–Fock [16] and configuration interaction (CI), the gradient can only be evaluated numerically so far, by finite differences of total energies. The lack of fast CI gradients is particularly restrictive for GUGA-CI [10] calculations of excited states, which constitute one of the most promising application areas for OMx methods.

The goal of this work is to derive and implement a *generic* semi-analytical approach to energy gradients for non-variational NDDO-type wavefunctions. The implementation will employ numerical integral and Fock matrix derivatives in the context of an otherwise analytical approach, and will therefore necessarily be less efficient than fully analytical techniques. On the other hand, this strategy leads to a great simplification of computer programming, and enables rapid implementation of fast semi-analytical gradients. The resulting code can almost trivially be extended to other NDDO-type Hamiltonians that might become available in the future.

The rest of this paper is organized as follows: Section 2 gives the generic expressions for CI energy derivatives in NDDO methods, and outlines their implementation in a

Dedicated to Professor Karl Jug on the occasion of his 65th birthday

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computer program. The asymptotic scaling of this implementation is examined and compared with fully numerical and fully analytical approaches. Section 3 discusses the numerical precision and computational performance of the new technique. Finally, Section 4 provides some conclusions and perspectives.

2 Methods and implementation

In all NDDO-type methods, the total CI energy of a system of nuclei and electrons is given by the sum of the core repulsion energy E_{core} and the electronic energy consisting of an SCF contribution E_{SCF} and a CI contribution E_{CI} .

$$E_{\text{tot}} = E_{\text{core}}(\{x_i\}) + E_{\text{SCF}}(\{x_i\}, \{P_{\mu\nu}\}) + E_{\text{CI}}(\{x_i\}, \{\Lambda_r\}, \{\Gamma_{pqrs}\}), \quad (1)$$

where $\{x_i\}$ are Cartesian coordinates of the nuclei, $\{P_{\mu\nu}\}$ are elements of the electron density matrix, $\{\Lambda_r\}$ are weights of one-electron states in the CI state of interest, and $\{\Gamma_{pqrs}\}$ are elements of the CI two-particle density matrix. The expression for the half-electron energy [8] can be cast into the same form. Because total energy expressions for NDDO-type methods are well documented elsewhere [4], we shall here only give the equations necessary to unambiguously establish the notation.

The core repulsion energy E_{core} does not depend (explicitly or implicitly) on the electronic structure. It is understood to include any constant reference energies or force-field corrections that might be introduced in a particular approach. The SCF contribution E_{SCF} represents the electronic energy of the variationally optimized, single-determinantal reference state. This term is a quadratic function of the electron density matrix:

$$E_{\text{SCF}} = \frac{1}{2} \text{Tr} \{ (\mathbf{H} + \mathbf{F}) \cdot \mathbf{P} \}, \quad (2)$$

where Tr stands for the matrix trace operation. The one-electron Hamiltonian matrix \mathbf{H} is an explicit function of the nuclear coordinates. The elements of the Fock matrix \mathbf{F} are linear functions of the density matrix, so that:

$$F_{\mu\nu} = \frac{\partial E_{\text{SCF}}}{\partial P_{\mu\nu}}. \quad (3)$$

The one-particle density matrix in Eq. (2) is subject to the commutation and idempotency conditions:

$$\mathbf{F} \cdot \mathbf{P} - \mathbf{P} \cdot \mathbf{F} = 0, \quad (4)$$

$$\mathbf{P} \cdot \mathbf{P} - \mathbf{P} = 0, \quad (5)$$

where the NDDO approximation is assumed to hold for the overlap matrix: $S_{\mu\nu} = \delta_{\mu\nu}$.

Finally, the CI energy term E_{CI} collects all contributions which are not variational with respect to the SCF density matrix \mathbf{P} , or have non-trivial dependence on the two-particle density Γ [9]:

$$E_{\text{CI}} = \sum_r^{\text{active}} \Lambda_r \epsilon_r + \sum_{pqrs}^{\text{active}} \Gamma_{pqrs} (pq|rs), \quad (6)$$

where the coefficients Λ_r and Γ_{pqrs} do not explicitly depend on the nuclear coordinates; ϵ_r and $(pq|rs)$ denote the eigenvalues of the Fock matrix and the two-electron integrals in the basis of molecular orbitals (MOs), respectively.

Analytical differentiation of the closed-form core repulsion contribution to the total energy is a straightforward application of the chain rule. Alternatively, this derivative can be calculated using a finite-difference numerical expression. To remain consistent with the treatment of one- and two-electron integrals, we choose to evaluate this contribution numerically.

Differentiation of Eq. (2) with respect to a nuclear coordinate x_i gives [17]:

$$\frac{dE_{\text{SCF}}}{dx_i} = \frac{1}{2} \text{Tr} \left\{ \left(\frac{\partial \mathbf{H}}{\partial x_i} + \frac{\partial \mathbf{F}}{\partial x_i} \right) \cdot \mathbf{P} \right\}. \quad (7)$$

Contributions containing derivatives of the density matrix \mathbf{P} , which would arise upon formal differentiation of Eq. (2), vanish due to the idempotency condition (Eq. 5). The integral derivatives in Eq. (7) can be determined by numerical differentiation of the matrix $(\mathbf{H} + \mathbf{F})$, which can be provided by a minor modification of the Fock matrix routine.

The remaining ingredient in the total energy expression is the CI energy E_{CI} . Evaluation of the corresponding derivatives requires calculation of wavefunction response, obtained from the solution of Coupled-Perturbed Hartree-Fock (CPHF) equations. Using the Z-vector [18] approach, the derivatives can be expressed as [8,9]:

$$\frac{dE_{\text{CI}}}{dx_i} = \sum_{pqrs}^{\text{active}} \Gamma_{pqrs} \frac{\partial}{\partial x_i} (pq|rs) + \text{Tr} \left\{ \frac{\partial \mathbf{F}}{\partial x_i} \cdot \mathbf{Z} \right\}. \quad (8)$$

In Eq. (8), the first term is the ‘‘static’’ part of the derivative, and the second term is the ‘‘response’’ part. The derivatives of the two-electron integrals $\frac{\partial}{\partial x_i} (pq|rs)$ and the Fock matrix $\frac{\partial \mathbf{F}}{\partial x_i}$ can be evaluated by numerical differentiation. Because all NDDO-type methods neglect three- and four-center two-electron integrals, differentiation can be performed in memory, without resorting to secondary storage or integral-direct approaches. Finally, calculation of the Z-vector requires only knowledge of the two-electron integrals, MO coefficients, and CI state parameters (Λ_r and Γ_{pqrs}). The CPHF equations are solved using the approach described previously [8], without any modification of the computer program.

In combined quantum mechanical / molecular mechanical (QM/MM) calculations, the electrostatic QM/MM interactions usually involve fixed MM point charges. Since the MM atoms do not carry basis functions, they affect only the one-electron part of the Hamiltonian. For the MM atoms, the expression for the SCF gradient, Eq. (7), thus reduces to:

$$\frac{dE_{\text{SCF}}}{dx_i} = \text{Tr} \left\{ \frac{\partial \mathbf{H}}{\partial x_i} \cdot \mathbf{P} \right\}. \quad (9)$$

When evaluating the gradient for the QM atoms, the one-electron QM/MM electrostatic contributions are easily included in the derivatives appearing on the right-hand sides of Eqs. (7) and (8). The presence of an external point-charge field

affects the response part of the CI gradient indirectly, through the SCF orbital coefficients. Our current implementation of semi-analytical gradients covers external point charges in the manner outlined above. Therefore, like our previous fully analytical approach [9], it can be applied in QM/MM calculations with electrostatic embedding [19].

The semi-analytical gradient evaluation employs numerical computation of the integral and Fock matrix derivatives in Eqs. (7) and (8). It is straightforward to accomplish this task by central finite-difference techniques using the existing routines for calculating the one- and two-electron integrals and for building the Fock matrix with a given constant density. Such a naive implementation is inefficient, however, because most of the integrals are not changed when displacing one particular atom in order to compute a particular Cartesian gradient component by finite difference. It is clearly sufficient to calculate only the affected integrals, i.e., those involving basis functions at the displaced atom. In the case of NDDO-type methods with only one- and two-center contributions to the energy (e.g., MNDO, AM1, PM3, and OM1), the computational cost for each Cartesian gradient component will then only scale with the number of atoms (N_{at}), and the cost for all required numerical integral and Fock matrix derivatives will scale as $O(N_{\text{at}}^2)$. In the case of the more recent approaches with three-center orthogonalization corrections (OM2 and OM3), the corresponding overall effort will increase as $O(N_{\text{at}}^3)$; it should be noted, however, that all three-center energy contributions in OM2 and OM3 involve products of integrals that decrease exponentially with increasing distance, and therefore screening procedures can be applied to improve the scaling behavior in this part of the computation if needed.

We now address the overall asymptotic computational cost of an energy and gradient evaluation at the OM2-CI level, which represents the most demanding case (see above). In Table 1, we compare the cost of a fully numerical calculation, the current semi-analytical approach, and a hypothetical “optimal” analytical implementation (not yet available) that exploits properties of the low-level integrals to reduce the amount of work. The two possible asymptotic scaling regimes are:

- (A) The number of CI configurations (N_{CSF}) is small, and does not increase with the number of atoms (N_{at}).
- (B) The number of CI configurations is large, and increases linearly (or faster) with the size of the system.

In the first case (A), the major time-consuming steps are the evaluation of the integrals over the basis functions and of their derivatives (always needed) as well as matrix diagonalization (only in the fully numerical approach). The fully numerical OM2-CI gradient will thus scale as $O(N_{\text{at}}^4)$, whereas the analytical and semi-analytical approaches should exhibit an $O(N_{\text{at}}^3)$ scaling. An optimum analytical implementation will be faster because of a smaller pre-factor.

A different scaling behavior is found in the case B, which is realized for calculations on excited states in larger conjugated molecules. In this case, CI energy evaluation may

be dominated by the computation of the GUGA-CI coupling coefficients or by the iterative diagonalization of the CI matrix, which are essentially both of order $O(N_{\text{CSF}}^2)$ (formal scaling, worst case), so that the numerical CI gradient calculation scales as $O(N_{\text{CSF}}^2 N_{\text{at}})$. Both the generic semi-analytical and the optimal analytical routines scale as $O(N_{\text{at}}^3)$, and have the same pre-factor.

Overall, our generic semi-analytical implementation is thus expected to be much faster than the fully numerical evaluation under all circumstances, and to approach the performance of an optimal analytical implementation for larger CI spaces.

Finally, the memory requirements are the same for analytical and semi-analytical gradient evaluations, because the finite-difference calculations can utilize the same arrays as standard energy evaluations. Moreover, the results from the calculation at the first displaced geometry (core Hamiltonian, Fock matrix, and two-electron integrals) can be stored in the standard arrays for the corresponding derivatives, before combining them with the results at the second displaced geometry, which yields the needed derivatives.

3 Precision and performance

We have tested our implementation of generic semi-analytical gradients in SCF and CI calculations using six semiempirical methods (MNDO, AM1, PM3, OM1, OM2, and OM3). Here, we shall present only some of the results obtained. We shall focus on AM1 and OM2 as representative examples of NDDO-type methods without and with three-center energy contributions, respectively. Unless noted otherwise, we have not applied any screening for the three-center terms in OM2 (see above).

The first issue is the precision of the computed gradients. To avoid any significant errors from SCF convergence in the tests on precision, we have used tight SCF convergence criteria, 10^{-12} eV for the electronic energy and 10^{-12} for the SCF density matrix. In the fully numerical gradient calculation, the step size DELTA in Cartesian coordinates was varied between 10^{-2} and 10^{-6} Å. Likewise, in the semi-analytical approach, the Cartesian step size DSTEP for numerical computation of integral and Fock matrix derivatives was varied between 10^{-3} and 10^{-6} Å. Finally, the analytical gradient code [8,9] controls the numerical precision (e.g., for solution of the CPHF equations) by a global option DPREC which was varied between 10^{-4} and 10^{-8} au.

Taking the SCF gradient of ethylene as an example, we find that each of the three approaches to gradient evaluation converges for tighter (decreasing) values of DELTA, DSTEP, and DPREC, respectively. Convergence of better than 10^{-7} au (typically 2×10^{-8} au) is observed for the Cartesian gradient norm with DELTA = 10^{-4} Å, DSTEP = 10^{-4} Å, and DPREC = 10^{-6} au. More importantly, all available approaches converge to the same result when using tight convergence criteria: for the chosen input geometry of ethylene, an identical AM1-SCF Cartesian gradient norm of 0.05747541

Table 1 Computational complexity of OM2-CI calculations: comparison of numerical, semi-analytical, and optimal analytical gradients^a

SCF+CI energy		
1-electron Hamiltonian	$O(N^3)$	
2-electron part of the Fock matrix	$O(N_{\text{at}}^2 m^4)$	
Matrix diagonalization	$O(N^3)$	
Computation of CI coupling coefficients	$O(N_{\text{CSF}}^2)^{\text{b}}$	
Iterative diagonalization of CI Hamiltonian	$O(N_{\text{CSF}}^2)^{\text{b,c}}$	
Overall	$O(N^3) + O(N_{\text{CSF}}^2)$	
Gradient		
	Numerical	Semi-analytical and analytical
1-electron Hamiltonian derivatives ^d	$O(N^3)$	
2-electron integral derivatives ^d	$O(N_{\text{at}}^2 m^4)$	
Solution of the CPHF equations	$O(N^3)$	
Evaluation of the CI response contribution	$O(N_{\text{at}} N^2)$	
Overall	$O(N_{\text{at}} N^3) + O(N_{\text{at}} N_{\text{CSF}}^2)^{\text{e}}$	$O(N^3)$

^a $N = N_{\text{at}} m$ is the number of basis functions; N_{at} is the number of atoms; N_{CSF} is the number of CI configurations; m is the average number of basis functions per atom

^b Formal scaling, worst case

^c This estimate is based on the assumptions that one CI state is computed and that the number of Davidson iterations is independent of the size of the molecule or the dimension of the CI problem

^d Gradient terms appearing in Eqs. (7) and (8) are calculated independently in our semi-analytical implementation, so that all integral derivatives are evaluated twice. Without knowledge of the low-level structure of the Hamiltonian matrix elements, storage requirements needed to eliminate the second integral pass would exhibit undesirable cubic scaling with the number of atoms

^e An SCF+CI calculation is performed for each of the $6N_{\text{at}}$ displacements

Table 2 Computation times (seconds) for minimal-CI energy and gradient of condensed aromatic compounds on one processor of a 2.0 GHz dual-Opteron system with 8 Gb of main memory

	AM1				OM2		
	SCF+CI		CI gradient		SCF+CI	CI gradient	
	Energy	Analytical	Hybrid	Numerical	Energy	Hybrid	Numerical
C ₁₀ H ₈	0.01	0.01	0.02	0.51	0.01	0.19	0.68
C ₃₂ H ₁₄	0.17	0.11	0.53	25.10	0.17	3.48	30.22
C ₆₆ H ₂₀	1.29	0.71	3.91	330.33	1.43	24.99	390.77
C ₁₁₂ H ₂₆	6.87	3.10	17.70	2750.09	7.45	111.98	3293.88
C ₁₇₀ H ₃₂	27.93	11.30	58.72	15859.74	28.03	388.96	18879.33

au is obtained from independent numerical, semi-analytical, and analytical calculations. In the case of OM2-SCF, the fully analytical gradient has not yet been implemented, but the other two approaches again yield the same Cartesian gradient norm of 0.06893268 au and thus agree to within 10^{-8} au. These and other tests confirm that our implementation of generic semi-analytical gradients is correct.

The performance of the new code has been studied using the default options for precision: SCF convergence criteria of 10^{-9} eV for the electronic energy and 10^{-9} for the SCF density matrix, DELTA = 10^{-4} Å, DSTEP = 2×10^{-4} Å, and DPREC = 10^{-6} au (see above). We have focused on CI gradients which are expected to represent typical applications of the new code, and we have chosen the same test cases as in our previous work, namely minimal CI calculations on condensed aromatic hydrocarbons [9] and more extensive GUGA-CI calculations on n -acenes [10]. These two types of calculations correspond to the scaling regimes (A) and (B), respectively (see above).

Table 2 shows AM1 and OM2 timings for minimal CI calculations. Here, the time for the energy evaluation is dom-

inated by the SCF part since the minimal 3×3 CI treatment [9] only requires negligible effort. The fully analytical AM1-CI gradient is faster than the semi-analytical AM1-CI gradient (“hybrid” in Tables 2–5) by a factor of about 5, but shows the same scaling behavior for increasing molecular size. The fully numerical AM1-CI gradient is much slower, by more than two orders of magnitude for the larger systems. As expected from scaling considerations (see above), the fully numerical OM2-CI gradient is also much slower than its semi-analytical counterpart, by a factor of about 50 for the largest case (C₁₇₀H₃₂). Comparing the AM1 and OM2 timings, it is obvious that the energy evaluation, and consequently also the fully numerical gradient evaluation, require similar computational effort. The computation time for the semi-analytical CI gradient is larger in OM2 than in AM1 (here typically by a factor of 6), which reflects the need to compute the three-center energy contributions in OM2 that are absent in AM1. As pointed out before, this part of the OM2 calculation can be accelerated through screening techniques. For example, in the case of C₁₇₀H₃₂, the computation time for the semi-analytical OM2-CI gradient drops from 389 s

Table 3 Computation times (seconds) for 5–5' full-CI energy and gradient for the lowest ${}^1B_{2u}$ state of n -acenes on one processor of a 2.0 GHz dual-Opteron system with 8 GB of main memory

Molecule	Active π -MO space	Number of CSFs	AM1				OM2		
			SCF	CI	CI gradient		SCF	CI	CI gradient
			Energy	Energy	Analytical	Hybrid	Energy	Energy	Hybrid
C ₁₀ H ₈	5-5'	4816	0.01	2.92	0.04	0.05	0.01	2.91	0.22
C ₁₄ H ₁₀	5-5'	4816	0.01	2.96	0.07	0.10	0.02	2.87	0.50
C ₁₈ H ₁₂	5-5'	4816	0.03	2.85	0.11	0.17	0.04	2.90	0.96
C ₂₂ H ₁₄	5-5'	4816	0.06	2.87	0.17	0.31	0.06	2.90	1.66
C ₂₆ H ₁₆	5-5'	4816	0.10	3.00	0.24	0.51	0.10	2.96	2.62

Table 4 Computation times (seconds) for MR-CISD energy and gradient for the lowest ${}^1B_{2u}$ state of hexacene with active π -MO spaces of different size, calculated on one processor of a 2.0 GHz dual-Opteron system with 8 GB of main memory

Molecule	Active π -MO space	Number of CSFs	AM1				OM2		
			SCF	CI	CI gradient		SCF	CI	CI gradient
			Energy	Energy	Analytical	Hybrid	Energy	Energy	Hybrid
C ₂₆ H ₁₆	5-5'	358	0.10	0.16	0.25	0.51	0.10	0.16	2.61
C ₂₆ H ₁₆	7-7'	1754	0.10	0.72	0.59	0.88	0.10	0.67	2.99
C ₂₆ H ₁₆	9-9'	5522	0.10	3.14	1.48	1.73	0.10	3.00	3.86
C ₂₆ H ₁₆	11-11'	13480	0.10	10.63	3.51	3.68	0.10	10.54	5.77
C ₂₆ H ₁₆	13-13'	27986	0.10	32.65	7.83	7.96	0.10	29.26	10.05

Table 5 Computation times (seconds) for MR-CISD energy and gradient for the lowest ${}^1B_{2u}$ state of n -acenes with all π MOs in the active space, calculated on one processor of a 2.0 GHz dual-Opteron system with 8 Gb of main memory

Molecule	Active π -MO space	Number of CSFs	AM1				OM2		
			SCF	CI	CI gradient		SCF	CI	CI gradient
			Energy	Energy	Analytical	Hybrid	Energy	Energy	Hybrid
C ₁₀ H ₈	5-5'	362	0.01	0.06	0.04	0.06	0.01	0.06	0.23
C ₁₄ H ₁₀	7-7'	1770	0.01	0.60	0.19	0.22	0.02	0.60	0.62
C ₁₈ H ₁₂	9-9'	5548	0.03	2.83	0.74	0.85	0.04	2.80	1.62
C ₂₂ H ₁₄	11-11'	13503	0.06	10.57	2.59	2.72	0.06	10.27	4.06
C ₂₆ H ₁₆	13-13'	27986	0.10	32.65	7.83	7.96	0.10	29.26	10.05

to 303 s when neglecting tiny three-center terms by applying an overlap cutoff criterion of 10^{-10} , which changes the computed Cartesian gradient norm by only 2×10^{-8} au. It should be stressed, however, that even without such screening and in spite of the fact that the tests in Table 2 fall into the unfavorable scaling regime (A), the semi-analytical OM2-CI gradient is much more efficient than a fully numerical one, which is the only available alternative.

Moving to more extensive CI calculations [10] and hence to scaling regime (B), Table 3 gives AM1 and OM2 timings for full CI calculations on n -acenes using an n - n' active space with $n = 5$ highest occupied π MOs and $n' = 5$ lowest unoccupied π MOs, which gives rise to 4816 configuration state functions (CSFs) for the lowest ${}^1B_{2u}$ state (D_{2h} symmetry). Tables 4 and 5 list analogous timings for multi-reference MR-CISD calculations with single and double excitations for active spaces of increasing size (up to 27986 CSFs), for hexacene and a series of n -acenes, respectively. Timings for fully numerical CI gradient calculations are not presented in these tables, but they require $6N_{\text{at}}$ energy evaluations and will therefore be slower than a CI energy evaluation by approximately this factor (not taking into account that CI energy cal-

culations at distorted geometries with lower symmetry will take longer than those at the D_{2h} reference geometry). It is obvious at first sight that the time for the energy evaluation is dominated by the CI part in all these examples. The fully analytical computation of the AM1-CI gradient is generally much faster than the AM1-CI energy evaluation. This is also true for the semi-analytical AM1-CI gradient in almost all cases, except for the smallest MR-CISD calculations (see Tables 4 and 5). The semi-analytical AM1-CI gradient is slightly slower than its fully analytical counterpart, but the differences in the timings tend to vanish with increasing number of CSFs, as expected from general scaling considerations (see above): for example, in the MR-CISD calculations for hexacene with 27986 CSFs, the corresponding computation times are 7.96 and 7.83 s (see Table 4). The semi-analytical CI gradient is usually somewhat slower in OM2 than in AM1 (due to the need to evaluate the additional three-center terms), but the difference is not pronounced and diminishes with increasing CI spaces. More importantly, the semi-analytical OM2-CI gradient calculation takes less time than the OM2-CI energy evaluation for most of the examples in Tables 3–5, except for the smallest MR-CISD cases. This

confirms that this approach is indeed efficient in the scaling regime (B), as expected, and is clearly preferable to a fully numerical gradient evaluation, which requires $6N_{\text{at}}$ OM2-CI energy evaluations.

4 Conclusions

Generic semi-analytical energy gradients have been implemented for non-variational NDDO-type wavefunctions, using numerical integral and Fock matrix derivatives in the context of an otherwise analytical approach. The correctness and numerical precision of this approach have been established by comparisons with fully numerical and (if available) fully analytical gradient calculations. The semi-analytical CI gradients have been shown to be much faster than fully numerical CI gradients, and they are therefore the method of choice whenever a fully analytical CI gradient is not available due to the lack of analytical integral derivatives. For large CI spaces, the semi-analytical treatment even approaches the performance of a fully analytical treatment.

Applications of the methods with orthogonalization corrections (OM1, OM2, OM3) will benefit most from the present development. With the availability of an efficient semi-analytical CI gradient, these methods can now be employed in explorations of excited-state potential energy

surfaces and studies of photochemistry in larger organic molecules, and corresponding applications to enzymes will be possible in the framework of QM/MM methods.

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